To catheterise or not to catheterise?

An approach based on decision theory

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SUMMARY To determine whether or not patients require cardiac catheterisation before surgery a computer based mathematical model was constructed based on decision theory. The model was specifically applied to sick infants under 3 months of age with suspected coarctation of the aorta, and a three way sensitivity analysis was carried out to assess the effects on the model of changes in the probabilities that underlie the decision itself.

The optimal decision (that with the greater survival rate) was moved away from cardiac catheterisation to confirm the diagnosis towards operating without cardiac catheterisation by the following factors: a higher probability of survival of operation both in the presence and absence of coarctation; a higher probability of survival if there was no coarctation and no operation performed; a lower sensitivity of catheterisation; a greater incremental risk of operation resulting from previous catheterisation; and a higher relative risk of catheterisation in patients without as opposed to with coarctation. Factors that tended significantly to move the decision towards catheterisation to rule out coarctation rather than neither to operate nor to catheterise were: a lower risk of surgery for coarctation if present; a higher risk of failing to operate on a patient who had coarctation; a high specificity of cardiac catheterisation; a lower incrementation of surgical risk by previous cardiac catheterisation; and a lower relative risk of catheterisation if coarctation was absent.

In this institution, the model argues strongly against cardiac catheterisation in the great majority of sick infants with coarctation.

Considerable contention currently exists over the question of to what extent cardiac catheterisation and angiography can be dispensed with in the preoperative evaluation of patients for cardiac surgery. The key question is whether the risk of invasive investigation is justified by its greater accuracy compared with alternative non-invasive investigations. Yet this question is essentially unanswerable unless risk and accuracy can be expressed in the same units, for only then can they be usefully compared. The purpose of this study was to show that risk and accuracy can be formally measured against one another if a mathematical approach based on decision theory is used. To make the results more tangible, we selected a particular decision on whether to catheterise sick young infants with coarctation of the aorta, but as will be seen the approach is generally applicable to any decision about cardiac catheterisation or indeed to any preoperative decision that involves a diagnostic procedure that is accurate but not without risk.

Methods

GENERAL APPLICATION OF MODEL

The probabilities underlying the decision were that (i) the patient may (D+) or may not (D–) have the disease, (ii) may (C+) or may not (C–) have a cardiac catheterisation, (iii) may (T+) or may not (T–) have the disease confirmed by the cardiac catheterisation, (iv) may (O+) or may not (O–) have an operation, (v) and may (L+) or may not (L–) survive.

Fig. 1 shows a decision tree for the problem. As the decision tree branches, reading from left to right, so each branch ultimately ends in a terminal node, which represents the outcome.3–6 Next to each terminal
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1. **Decision tree for general model concerning the question of whether to catheterise a patient or not before operating on a particular defect.** See text and footnote to Table for explanation of symbols.

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**Node** is a utility, which for this particular problem is defined as zero for death and one for survival.

The first decision is either (i) to catheterise the patient and then decide whether or not to operate according to the result of the catheterisation, (ii) to operate without cardiac catheterisation, or (iii) neither to operate nor to carry out a cardiac catheterisation. The branch corresponding to each of these three decisions ends in a probability node. The definitions of the probabilities used in Fig. 1 are given in the Table. Thus for the decision neither to operate nor to carry out cardiac catheterisation, the probability node has two branches, one corresponding to the probability that the patient has the disease (p) and the other to the probability that he does not (1−p). If the patient has the disease there is a probability c that he will survive, and 1−c that he will die. If, on the other hand, he does not have the disease the probabilities that he will live and die are d and 1−d respectively. Both c and d are conditional probabilities—namely, probabilities of an event given that another event or events has occurred. Furthermore, for each probability node the possibilities are mutually exclusive and exhaustive, which means that the sum of the probabilities must equal one.

**Operate without catheterisation**

The decision to operate without cardiac catheterisation does not change the probability that the disease is present but does change the probabilities of survival, since the patient has now been operated on. If the patient does have the disease and is operated on, it is the probability of survival. On the other hand, if the patient does not have the disease and is operated on the probability of survival is b.

**Catheterisation**

Again the decision to catheterise does not affect the probability that the disease is present. Nevertheless, the patient may die either during cardiac catheterisation or between cardiac catheterisation and implementation of the decision on whether to operate or not. Because our objective is to focus on the risk involved in cardiac catheterisation, we defined the probability of death as a result of cardiac catheterisation as r, given that the disease was present. It could be argued that if the disease were absent the risk of catheterisation would be different. To allow for this contingency in the simplest possible way, we introduced a constant of proportionality (h), which represents the ratio between the risk of cardiac catheterisation if the disease is absent and the risk if it is present. If the patient survives catheterisation, there is still a possibility that cardiac catheterisation will give inaccurate information. Given that the disease is present, the probability of the catheterisation confirming the disease—that is, its sensitivity—is e. On the other hand, given that the disease is absent, the probability that catheterisation will confirm the disease is f (1—the sensitivity of catheterisation for this disease).

The decision tree from e, (1−e), f, and (1−f) has four decision nodes. These correspond to the decision on whether to operate or not depending on the results of cardiac catheterisation. We have assumed that if catheterisation confirms the disease, we will invariably decide to operate, whereas if it shows that the disease is not present we will decide not to. Consequently, from each decision node springs only one decision. Nevertheless, it is possible to have a second branch coming from each of these decision nodes and to consider the risks of operating or not operating accordingly. If this is done for coarctation in sick young infants, the decision model confirms that it is...
invariably correct to decide to operate if cardiac catheterisation shows that the patient has a coarctation and not to operate for coarctation if catheterisation shows that he does not. In short, the branches corresponding to doing the opposite of what the cardiac catheterisation suggests have been “pruned.”

If the decision is not to operate the probabilities of survival and death are assumed to be equal to those pertaining to the decision neither to catheterise nor to operate. In other words, probabilities c and d remain unchanged. On the other hand, it may be that, although cardiac catheterisation does not result in death before operation is carried out, it increases the risk of the operation by virtue of the deterioration in the patient’s general condition. To represent this increase in risk, we introduced a second constant of proportionality g, which represents the ratio between the probability of surviving the operation in the presence of the disease if the patient has been catheterised and the corresponding probability if the patient has not.

**Computation of survival chances**

According to classical decision theory, the best decision to make is that which maximises expected utility, which in this application is that which maximises the chance of survival. This is obtained by summing the products of probabilities and utilities all along the branches corresponding to a particular decision. Since just over half the branches terminate in death, which has a utility of 0, multiplying any probability by that utility will result in an answer of 0, thus easing computation considerably. We applied this method to each branch of the decision tree in turn, starting with the two decisions which involve not catheterising the patient.

**Expected utility of operation without cardiac catheterisation**

\[ E(u(C+)) = p(a-b-c+d) = d-b \]

**Expected utility of neither operating nor catheterising**

\[ E(u(C-)) = p(c-d)+d \]

When these two expected utilities are equal, one decision is as good as the other. This equilibrium point occurs at what is termed the critical probability. Setting these expected utilities equal and solving for the critical probability, we obtain:

\[ p = \frac{d-b}{a-b-c+d} \]

**Expected utility of cardiac catheterisation**

\[ E(u(C+)) = p(1-r) \{ e+g+(1-e)c \} + (1-p) (1-r) \{ b+f+(1-f)d \} \]

Let \( x = \{ e+g+(1-e)c \} \) and \( y = \{ b+f+(1-f)d \} \)

\[ E(u(C+)) = px - rpx + y + rhyp - py - rhy \]

\[ = r(hyp - px - hy) + y + p(x-y) \]

Let \( z = hy - px - hy = (hy - x) - hy \)

Then \[ E(u(C+)) = rz + y + p(x-y) \]

Setting equal the expected utilities of operating without cardiac catheterisation and cardiac catheterisation we obtain:
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(2) \( rz + y + p(x - y) = p(a - b) + b \)

Whence \( r = \frac{p(a - b - x + y) + b - y}{p(hy - x) - hy} \)

Similarly, setting equal the expected utilities of neither operating nor catheterising, and catheterising, we obtain:

(3) \( r = \frac{p(c - d - x + y) + d - y}{p(hy - x) - hy} \)

If we plot \( r \) against \( p \) the above three equations give three lines corresponding to critical probabilities separating the three decisions producing the greater expected utility (Fig. 2). That corresponding to equation (1) is not dependent on \( r \) and therefore corresponds to a vertical line separating the decision to operate or not to operate, in neither case performing cardiac catheterisation. That line corresponding to equation (2) separates the decision to operate without cardiac catheterisation from the decision to catheterise. The line corresponding to equation (3) separates the decision neither to operate nor catheterise from the decision to catheterise. All three lines meet at the equilibrium point when each decision has the same expected utility. In Figs. 3-6 the vertical line ascending from the equilibrium point has been omitted for the sake of clarity. Its position can be easily imagined, since it is always vertical and always originates from the apex of the triangle.

![Graph](http://heart.bmj.com/)

**Fig. 2** General form of the solution of the equations for critical probability produced by the model. Three lines meet at the apex of a triangle at the base of the graph. This triangle corresponds to the decision to catheterise. The area above and to the right of the triangle corresponds to the decision to operate without cardiac catheterisation, and the area above and to the left of the triangle corresponds to the decision neither to operate nor to perform cardiac catheterisation.

COARCTATION OF THE AORTA

The model was also applied to the particular problem of deciding whether or not to catheterise a baby in heart failure in the first three months of life who may have coarctation of the aorta. The Table lists the values of probabilities chosen for this particular problem, which correspond to our experience at The Hospital for Sick Children, London. Ideally, all these probabilities should be based on long run experiments, but these have not been used for the following reasons. Firstly, our objective was to demonstrate the principles that underlie rational decision making rather than to show what particular decision is correct in this particular condition. Secondly, long run probabilities collected in one particular centre will almost certainly not correspond to long run probabilities collected from another. Thirdly, some of these probabilities are by their nature either extremely difficult or impossible to obtain by long run experiments. For example, it would be virtually impossible to establish what the increase of risk of operation produced by previous catheterisation is, because the effect of that particular factor would be so easily obscured by all the other factors that affect the risk of operation. Fortunately, decision theory does not require that these probabilities be precisely estimated on the basis of long run experiments. Subjective probabilities, based on clinical intuition, are perfectly acceptable, providing that careful sensitivity analysis clearly shows the effect of perturbing the subjective probabilities within clinically reasonable bounds.

To investigate the effects of changing the subjectively assessed variables of the model, we performed a sensitivity analysis by programming an Apple II Euro microcomputer to solve the above equations and plot the result interactively.

**Results**

Using the computer model as many of the variables may be varied simultaneously as required, thereby obtaining an infinitely large number of results. For simplicity, we present the results of changing one variable at a time while holding the others constant. The resulting figures thus show how the optimal decisions depend on the value of \( p \), \( r \), and the variable of interest and are known as a "three-way sensitivity analysis." The meaning of \( r \), the mortality of cardiac catheterisation, on the ordinate of Figs. 2-6 is self explanatory. The abscissa, corresponding to the probability \( p \) of the disease being present, represents the probability at any stage in the diagnostic procedure before cardiac catheterisation. Nevertheless, for the particular purposes of assessing the need for cardiac catheterisation, this probability represents the summation of all the non-invasive information that can be
obtained from the patient. In Figs. 2–6 there is a roughly triangular area at the base of the graph corresponding to the decision to catheterise. At the right hand end of the triangle, corresponding to high values of probability that a coarctation is present, if catheterisation is recommended this is so as to confirm the suspected diagnosis. Contrariwise, at the left hand end of the triangle, corresponding to low probabilities of diagnosis of coarctation, cardiac catheterisation is being recommended (if it is) to rule out the diagnosis of coarctation. The greater the total area of the triangle, the more desirable is cardiac catheterisation on average. Shift of the equilibrium point to the left indicates readiness to operate for coarctation when there is less certainty of its existence. Shift of the right hand limb of the triangle upwards and to the right favours catheterisation to confirm the diagnosis of coarctation, whereas shift of the left hand limb of the triangle upwards and to the left favours cardiac catheterisation to rule out the diagnosis. Because the right hand end of the triangle is of more immediate practical interest, it has been enlarged in the right upper quadrant of each graph in Figs. 3–6.

ALTERATIONS IN PROBABILITY OF SURVIVAL

If disease present and operated on

This amounts to the operative risk of surgery without cardiac catheterisation. As can be seen from Fig. 3a the higher the risk of surgery, the more desirable is cardiac catheterisation to confirm the diagnosis, whereas the less desirable is cardiac catheterisation to rule out the diagnosis. This makes intuitive sense. The more successful the operation is, the less impor-
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...it is to be absolutely sure that the disease is present before operating, and the more important it is to be as sure as possible that the presence of an eminently treatable condition is ruled out. The equilibrium point is moved to the left by the high probabilities of surviving surgery, since if operation is highly likely to be successful, one needs to be less certain that the condition is present before operating.

If no disease but operation performed
In this instance an inappropriate operation has been carried out (Fig. 3b). Since babies in the first three months of life, who are in heart failure and yet who do not have coarctation are likely to have another serious cardiac condition, the probability of survival after an inappropriate operation has been deliberately set rather low at 0.6. Clearly, if the penalty for doing an inappropriate operation is lower—that is, the probability of survival from such an operation is higher—then this favours non-invasive investigation, since there is less need for the accuracy provided by cardiac catheterisation. Changes in this variable do not affect the decision on whether to catheterise to rule out the diagnosis of coarctation.

If disease present but not operated on
Changing this variable has a marginal effect on the desirability of cardiac catheterisation to confirm the diagnosis of coarctation (Fig. 3c). On the other hand, it has quite a considerable effect on carrying out cardiac catheterisation to rule out the diagnosis. If the probability of survival is very low if coarctation is present and no operation is performed it is clearly important to be as sure as possible that there is no coarctation.

If no disease and no operation performed
Patients in heart failure within the first three months of life who do not have coarctation of the aorta probably have a ventricular septal defect. This is why the baseline probability of survival has been set as high as 0.95 (Fig. 3d). The major effect of changing this variable is on the decision to catheterise to confirm the diagnosis of coarctation. If the natural history of the alternative diagnoses is better then it becomes less important to be accurate about the diagnosis of coarctation.

SENSITIVITY OF CARDIAC CATHETERISATION
If there is a communication at ventricular level, with shunting through it, then contrast medium injected into the ventricle can opacify the ascending and descending aorta, as well as a ductus, simultaneously. Under these circumstances, the diagnosis of coarctation may be missed. We set the sensitivity of catheterisation at 0.99 (Fig. 4a). Clearly, if the sensitivity is higher this makes cardiac catheterisation more desirable, but even for a sensitivity of 0.999, given the other variables, we have to be only 93% certain that coarctation is present to recommend operation without cardiac catheterisation even if cardiac catheterisation carries no mortality at all. Changing the sensitivity of cardiac catheterisation clearly has little effect on its usefulness in ruling out the diagnosis. In contrast to the first four examples (Fig. 3), changing the variables specifically associated with cardiac catheterisation does not shift the equilibrium point sideways, since it has no effect on the decision on whether to operate or not if no cardiac catheterisation is carried out.

SPECIFICITY OF CARDIAC CATHETERISATION
The specificity of cardiac catheterisation is not 100% because of the existence of pseudocoarctation. We set the baseline specificity at 98% (Fig. 4b). Clearly, if
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Fig. 5  Effect on the model of alterations in the relative reduction of probability of survival of operation in the presence of coarctation owing to cardiac catheterisation having been carried out. For a basic 90% chance of survival if operation is carried out for coarctation of the aorta without previous cardiac catheterisation, the values of 0.95, 0.97, and 0.99 correspond to incrementation of the risk of operation by 4.5%, 2.7%, and 0.9%.

Fig. 6  Effects on the model of changing the relative risk of cardiac catheterisation in the absence of coarctation compared with that in its presence.

this is higher catheterisation is more desirable to rule out the diagnosis of coarctation. Changing the specificity of catheterisation has relatively little effect on its desirability to confirm the diagnosis.

INCREMENTATION OF RISK OF OPERATION BY PREVIOUS CATHETERISATION

Fig. 5 shows that if catheterisation produces an incrementation in the risk associated with operation for coarctation then it is less desirable if coarctation is highly likely to be present. If coarctation is unlikely to be present then operation is unlikely to take place and the effect will be marginal.

Relative risk between absence and presence of disease

Fig. 6 shows that if the relative risk of catheterisation in the absence of the disease compared with that in its presence is lower, whether a coarctation is likely to be present or not, catheterisation is more desirable. This is because the overall risk of catheterisation for all conditions including coarctation will be lower. Nevertheless, the changes produced are very minor at the two ends of the triangle, which correspond to the points of greatest interest to the clinician.

Discussion

The concept of decision theory is not new in clinical medicine, and has been applied sporadically in both acquired and congenital heart disease. We have, however, found that its application is particularly well suited to the question of the necessity for cardiac catheterisation for three particular reasons. Firstly, the problem is not esoteric; it is faced daily by cardiologists. Secondly, it is relatively straightforward and thereby unlikely to daunt the mathematically unsophisticated. Thirdly, it shows very clearly that a rational approach to the question demands recognition that what appears on the surface to be a straightforward matter actually involves the subtle interaction of at least the eight different probabilities or ratios of probabilities that we have considered.

Simplistic approaches to the question, such as “We must have perfectly accurate information before proceeding with cardiac surgery” or conversely “No patient of mine is going to fall into the hands of a catheterizer,” are obviously inadequate but so too are less extreme positions which, nevertheless, fail to take into account the whole range of uncertainties involved. One great merit of decision theory is that even if no mathematical calculations are made, construction of the decision tree provides a rigorous intellectual framework for analysing the problem. If the logic of that decision tree is accepted, and the appropriateness of the variables is also acknowledged, then it is difficult to argue with the conclusion.

ADEQUACY OF THE MODEL

The model was deliberately made as simple as possible for the sake of clarity. It may be argued that the proportionality variables g and h are inappropriate. If so, the incrementation of risk of surgery by previous catheterisation, and of catheterisation by disease, can be easily dealt with by having more branches on the decision tree.
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Does the decision tree apply to all patients with suspected coarctation? What about patients with associated tricuspid atresia or transposition of the great arteries? We estimated our baseline variables on the assumption that we were dealing with the entire population of infants with coarctation, but there is no reason why the same decision tree should not be used to assess the desirability of catheterising diagnostic subgroups. In most cases all that needs to be changed is some of the variables, although in other cases, such as patients with complete transposition with coarctation, the decision tree would require modification because of the question of carrying out balloon atrial septostomy.

ADEQUACY OF THE VARIABLES

As has been explained, it is desirable where possible to use long run probabilities in the decision model since these are more objective. Where the value of variables changes with time, however, the use of long run probabilities also carries inherent disadvantages. To apply the decision model correctly to a patient today, we need, for example, to know what is the probability of surviving an appropriate operation for coarctation today, not what it was 10 years ago nor even what it was on average over the past 10 years. Statistical methods do exist for obtaining an estimate of risk today from a study based on experience over a long time (for example, including the era of operation as a risk factor in stepwise multiple logistic regression), but the necessary information to obtain even this contemporary probability is almost entirely lacking from published reports. This is despite the fact that an estimate of the probability of hospital survival of operation is the one piece of information that can be guaranteed to be found in any surgical report on the management of the disease concerned. If the paucity of really useful information applies to such a widely publicised statistic what chance is there of obtaining objective data on such imponderables as the risk of cardiac catheterisation in a disease like coarctation of the aorta? If the estimated risk is based on whether the patient leaves the catheterisation laboratory alive, the major determinant of apparent mortality may well be how fast the patient can be transferred out of the catheterisation laboratory. If, on the other hand, the conventional definition of mortality within 24 hours of the onset of cardiac catheterisation is used (as in the New England Regional Infant Cardiac Program study), where 24 hour mortality was 6%, then this figure may depend largely on how quickly after catheterisation the patient is operated on and how good the surgeons are. When the frailty of such long run probabilities are considered, the subjective estimate of an expert clinician begins to look quite attractive. By using three way sensitivity analysis, we have shown the effect on the model of as wide a range of probabilities as most clinicians would probably regard as reasonable, but if not the model may be rerun with other variables.

LESSONS TO BE LEARNT FROM THE MODEL

In presenting our results, we have been careful to show that they are consistent with what clinical intuition would suggest. We must, however, emphasise that for some of the variables, if at the beginning we had guessed how they would affect the model, we might have either found the task impossible or have predicted something other than what is actually observed. This applies particularly to the effect of altering the probability of survival if neither operation nor cardiac catheterisation is carried out. Construction of the decision model has, therefore, been instructive. Another example is the finding that the better the results of operation for coarctation the less cardiac catheterisation is required to confirm the diagnosis. This is perhaps surprising. This effect persists even when previous cardiac catheterisation is not held to increment the risk of surgery at all.

Finally, in relation to the specific decision on whether patients with suspected coarctation in the first three months of life require cardiac catheterisation, we have recently completed a study on non-invasive diagnosis at this age. Using a combination of analysis of the distal peripheral pulses and cross sectional echocardiography it appears that one can be 93-8% certain of the presence of coarctation on the basis of these two investigations alone. Given that the risk of cardiac catheterisation alone (ignoring incrementation of surgical risk) must be 2% at minimum, inspection of Figs. 3 to 6 will show very clearly why our present policy is to catheterise hardly any babies with coarctation at all in the first three months.

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References

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